Total number of printed pages–6

3 (Sem–4 /CBCS) MAT HC 1

2021

MATHEMATICS

(Honours)

Paper : MAT–HC–4016

(Multivariate Calculus)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

Group – A

Marks : 40

1. Answer the following questions : 1×6=6

(i) Describe the domain of

$$f(x, y) = \sqrt{9 - x^2 - 4y^2}$$

(ii) If $f(x, y) = \sin\left(x^2 \cos y\right)$, determine f_x.

Contd.

(iii) When the function $f(x,y)$ is said to have relative maximum at the point (x_0, y_0) of its domain ?

(iv) State Green's theorem.

(v) What is the *curl* of a constant vector field ?

(vi) When the divergence and *curl* both are zero for a vector field ?

2. Answer the following questions : 2×5=10

(a) Show that the function $f(x,y) = e^y \cos x$ is harmonic.

(b) Find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$, when $x = u + 2v,\ y = 3u - 4v$.

(c) Use iterated integration to compute $\iint_R x^2 y\,dA,\ R: 1 \le x \le 2,\ 0 \le y \le 1$.

(d) Find gradient of the function $f(x,y) = x^2 y + y^3$.

(e) How the directional derivative $D_u f(x,y)$ of a function $f(x,y)$ can be expressed in terms of the gradient of $f(x,y)$?

3. Answer ***any four*** questions : 6×4=24

(a) Sketch the level curves of the function $f(x,y)=C$ for $C = 1, 4$ where $f(x, y) = x^2 + \frac{y^2}{4}$.

(b) Show that $\lim_{(x, y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$ does not exist.

(c) Find the equation for the tangent plane to the surface $z = tan^{-1}\left(\frac{y}{x}\right)$ at the point $P_0\left(1, \sqrt{3}, \frac{\pi}{3}\right)$.

(d) Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$, where $w = e^{2x-y+3z^2}$ and $x = r+s-t,\ y = 2r-3s,\ z = \cos(rst)$.

(e) Use the method of Lagrange's multipliers to minimize $f(x, y) = xy$ subject to $2x+2y=5$.

(f) Evaluate $\int_0^3 \int_0^{\sqrt{9-x^2}} x\,dy\,dx$ by converting to polar coordinates.

Group – B

Marks : 40

Answer the following questions : ***(any four)***

10×4=40

4. What is meant by simply-connected region? State Green's theorem for a simply-connected region. Verify Green's theorem for the line integral $\oint_C (-y\,dx + x\,dy)$, where C is the closed path consisting of the line segment from (–1,0) to (1,0) followed by the semicircular arc from (1,0) back to (–1,0).

5. Define the line integral $\int_C f ds$. What is the difference between $\int_C f ds$ and $\oint_C f ds$.

Evaluate the line integral $\int_C xy ds$, where C consist of the line segment C_1 from (–3,3) to (0,0), followed by the portion of the curve $C_2 : 16y = x^4$ between (0,0) and (2,1).

6. Define critical point of a function f defined on an open set D. Find all the critical points on the graph of $f(x,y) = 8x^3 - 24xy + y^3$ and use the second partial test to classify each point as a relative extremum or a sadle point.

7. Find the volume of the solid in the first octant that is bounded by the cylinder $x^2 + y^2 = 2y$, the half cone $z = \sqrt{x^2 + y^2}$, and the xy-plane.

8. Find the surface area of that part of the paraboloid $x^2 + y^2 + z = 5$ that lies above the plane $z = 1$.

9. Let (x, y, z) lie on the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. Without solving for z explicitly in terms of x and y, compute the higher order derivatives $\frac{\partial^2 z}{\partial x^2}$ and $\frac{\partial^2 z}{\partial x \partial y}$.
