3 (Sem–5) MAT M 3

2018

MATHEMATICS

(Major)

Paper : 5.3

(Spherical Trigonometry and Astronomy)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : $1\times7=7$

(a) State one fundamental difference between a spherical triangle and a plane triangle.

(b) Define polar triangle and its primitive triangle.

(c) Mention one property of pole of a great circle.

(d) What is the reason of the oval shape of the sun at rising?

(e) Explain briefly the dynamical significance of Kepler's second law of motion.

(f) Define orbital period and synodic period of a planet.

(g) What is the declination of the pole of the ecliptic?

(Turn Over)

2. Answer the following questions : 2×4=

(a) Drawing a neat diagram, discuss how horizontal coordinates of a heavenly body are measured.

(b) Prove that section of a sphere by a plane is a circle.

(c) Show that right ascension α and declination δ of the sun is always connected by the equation

$$\tan\delta = \tan\varepsilon \sin\alpha$$

ε being obliquity of the ecliptic.

(d) The apparent altitude of a star due to refraction is 30°. Calculate the true altitude, the coefficient of refraction being 58·2″.

3. Answer any *three* questions of the following : 5×3=1

(a) A port is in latitude l (north) and longitude λ (west). Show that the longitudes of places on the equator distance δ from the port are

$$\lambda \pm \cos^{-1}(\cos\delta \sec l)$$

(b) What do you mean by rising and setting of a star? Prove that the hour angle H of a star at the time of setting is given by

$$\cos H = -\tan\phi \tan\delta$$

(Continued)

(c) Prove that

$$\cos v = \frac{\cos E - e}{1 - e\cos E} \text{ and } \sin v = \frac{\sqrt{1-e^2}\sin E}{1 - e\cos E}$$

where v is the true anomaly and E is the eccentric anomaly at any position of a planet in its orbit.

(d) If λ is the moon's celestial latitude at the instant of opposition, m and p her hourly motions in longitude and latitude respectively, s the hourly motion of the sun in longitude and C the sum of semi-diameters of the moon and that of the earth's shadow, show that the duration of the lunar eclipse is the difference between the two roots of t, given by

$$C^2 = (\lambda - pt)^2 + (m-s)^2 t^2$$

(e) Define geocentric parallax. Show that geocentric parallax of a heavenly body varies as the sine of its apparent zenith distance.

4. Derive cosine formula related to a spherical triangle. In an equilateral spherical triangle ABC, prove the following : 6+4=10

(i) $2\cos\frac{a}{2} \cdot \sin\frac{A}{2} = 1$

(ii) $\sec A = 1 + \sec a$

(Turn Over)

5. (a) Derive the formula for refraction

$$R = k\tan\zeta$$

ζ being the apparent zenith distance of a heavenly body. Mention one limitation of this formula. 5+1=

(b) If z_1 and z_2 are the zenith distances of a star at upper and lower culmination respectively which are on opposite sides of the zenith, prove that

$$\delta = 90° - \frac{z_1 + z_2}{2} \text{ and } \phi = 90° - \frac{z_2 - z_1}{2}$$

where δ is the declination of the star and ϕ is the latitude of the place of observer.

6. Define solar ecliptic limits. Show that the minimum angular distance D_0 of the moon and the sun for occurrence of solar eclipse will be

$$D_0 = \beta\cos j$$

where $\tan j = \dfrac{\tan i}{1-m}$ the other symbols carry usual meanings. 2+8=1

Or

Discuss the effects of annual parallax on celestial longitude and latitude. 1

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