2012

MATHEMATICS

(Major)

Paper : 2.2

(Differential Equation)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following : 1×10=10

(a) Determine the order, degree, unknown functions and independent variable of the differential equation

$$5\left(\frac{d^4b}{dp^4}\right)^5 + 7\left(\frac{db}{dp}\right)^{10} + b^7 - b^5 = p$$

(b) Which of the following functions is solution of the differential equation $y_2 - y = 0$?

(i) $y = e^x$

(ii) $y = \sin x$

(iii) $y = 4e^{-x}$

(iv) $y = \tan x$

(c) When the first-order and first-degree differential equation

$$M\,dx + N\,dy = 0$$

(M and N are functions of x and y) is said to be exact?

(d) Write down the general solution of the differential equation

$$y = px + \sqrt{a^2 p^2 + b^2}, \quad p = \frac{dy}{dx}$$

(e) Find an integral belonging to complementary function of the differential equation

$$x^2 y_2 - 2x(1+x)y_1 + 2(1+x)y = x^3$$

(f) Write down the necessary condition for integrability of single differential equation

$$Pdx + Qdy + Rdz = 0$$

(g) Choose the key giving the correct answer :

The partial differential equations can be formed by the elimination of

(i) arbitrary constants only

(ii) arbitrary functions only

(iii) arbitrary functions or arbitrary constants

(iv) None of the above

(h) Write down the particular integral of the differential equation

$$y_2 + 3y_1 + 2y = e^x$$

(i) Give the geometrical interpretation of the differential equation

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

(P, Q and R are functions of x, y and z)

(j) Construct the partial differential equation by eliminating a and b from

$$z = ax + (1-a)y + b$$

2. Answer the following : 2×5=10

(a) Determine c_1 and c_2 so that

$$y(x) = c_1 e^{2x} + c_2 e^x + 2\sin x$$

will satisfy the condition $y(0) = 0$ and $y'(0) = 1$.

(b) Solve :

$$(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0$$

(c) Solve :

$$\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$$

(d) Solve :

$$\frac{dx}{dt} = x^2 - 2x + 2$$

(e) Find the differential equation of the family of curves $y = Ae^{3x} + Be^{5x}$ for different values of A and B.

3. Answer any *four* parts : 5×4=20

(a) Find the orthogonal trajectories of the family of curves

$$\frac{x^2}{a^2} + \frac{y^2}{b^2 + k} = 1$$

k being the parameter.

(b) Solve by the method of variation of parameter :

$$y_2 - y = \frac{2}{1 + e^x}$$

(c) Solve

$$\left(\frac{dy}{dx}\right)(x^2y^3 + xy) = 1$$

given $y = 0$ when $x = 1$.

(d) Find a partial differential equation by eliminating the arbitrary function ϕ from

$$\phi(x + y + z, x^2 + y^2 - z^2) = 0$$

(e) Solve the differential equation

$$\sin^2 x\left(\frac{d^2y}{dx^2}\right) = 2y$$

given that $y = \cot x$ is a solution.

(f) Show that the differential equation of all cones which have their vertices at the origin is $px + qy = z$. Verify that $yz + zx + xy = 0$ is a surface satisfying the above equation.

4. Answer *either* (a) and (b) *or* (c) and (d) : 5+5

(a) Solve :

$$(D^2 + 3D + 2)y = e^{2x} \sin x$$

(b) Solve :

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$$

(c) Solve :

$$p^2 + 2py\cot x = y^2$$

(d) Show that the following equation is exact and hence solve it :

$$\left\{y\left(1 + \frac{1}{x}\right) + \cos y\right\} dx + \{x + \log x - x\sin y\}\, dy = 0$$

5. Answer *either* (a) *or* (b) and (c) : 5+5

(a) Reduce the differential equation

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x)$$

to the form $\dfrac{d^2v}{dx^2} + Iv = S$

(I, S are functions of x) to solve the differential equation. Hence solve the following equation :

$$y_2 - 4xy_1 + (4x^2 - 3)y = e^{x^2}$$

(b) If $y = y_1(x)$ and $y = y_2(x)$ are two solutions of the equation

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$$

prove that

$$y_1 \frac{dy_2}{dx} - y_2 \frac{dy_1}{dx} = ce^{-\int Pdx}$$

c being an arbitrary constant.

(c) Solve the differential equation

$$x^6 y'' + 3x^5 y' + a^2 y = \frac{1}{x}$$

by changing the independent variable x to z.

6. Answer *either* (a) and (b) *or* (c) and (d) : 5+5

(a) Find $f(y)$ such that the total differential equation

$$\left(\frac{yz + z}{x}\right) dx - zdy + f(y)\, dz = 0$$

is integrable. Hence solve it.

(b) Solve :

$$xz^3 dx - zdy + 2ydz = 0$$

(c) Solve :

$$\frac{dx}{dt} + 5x + y = e^t$$
$$\frac{dy}{dt} - x + 3y = e^{2t}$$

(d) Solve the simultaneous equations :

$$\frac{adx}{yz(b-c)} = \frac{bdy}{zx(c-a)} = \frac{cdz}{xy(a-b)}$$

7. Answer *either* (a) and (b) *or* (c) and (d) : 5+5

(a) Solve by Lagrange's method :

$$p + q = x + y + z$$

(b) Find the integral surface of the partial differential equation

$$(x-y)p + (y-x-z)q = z$$

through the circle $z = 1,\ x^2 + y^2 = 1$.

(c) Solve by Charpit's method :

$$(p^2 + q^2)y = qz$$

(d) Find the complete integral of

$$z^2(p^2 z^2 + q^2) = 1$$

Find also the singular integral if it exists.

★★★