2012

MATHEMATICS

(Major)

Paper : 1.2

(Coordinate Geometry and Differential Equations)

Full Marks : 90

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Coordinate Geometry)

(*Marks* : 54)

1. *(a)* If by an orthogonal transformation without change of origin, the equation $ax^2 + 2hxy + by^2 = c$ is changed into an equation without xy term, show that it is $(a+b+\lambda)x^2 + (a+b-\lambda)y^2 = 2c$ where $\lambda = \sqrt{(a-b)^2 + 4h^2}$. 4

(b) Prove that by orthogonal transformation without changing the origin, the quantity $g^2 + f^2$ in the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

is an invariant. 4

2. *(a)* Show that the equation of the pair of tangents to the parabola $y^2 = 4ax$ can be expressed as $SS' = T^2$ with meanings of the symbols. 6

(b) Show that the polar of any point on the circle $x^2 + y^2 - 2ax - 3a^2 = 0$ with respect to the circle $x^2 + y^2 + 2ax - 3a^2 = 0$ will touch the parabola $y^2 + 4ax = 0$. 6

Or

Prove that the area of the parallelogram formed by the tangents at the ends of a pair of conjugate diameters is constant.

(c) Find the equation of pair of diameters conjugate with respect to both the conics $ax^2 + 2hxy + by^2 = 1$ and $a'x^2 + 2h'xy + b'y^2 = 1$. 6

Or

Find the equation of the hyperbola having $y - mx = 0$ and $y + mx = 0$ as asymptotes and passing through (1, 0).

3. *(a)* Find the distance of the point (1, 2, 3) from the plane $2x + 3y + 4z = 25$ measured parallel to the line

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$$

What is the peculiarity of this distance? 5

(b) Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y+z=0,\ z+x=0,\ x+y=0,\ x+y+z=a$ is $2a/\sqrt{6}$; and that the three lines of shortest distances intersect at the point $x=y=z=-a$. 5

4. (a) Prove that the tangent planes to the spheres

$$x^2+y^2+z^2+2u_ix+2v_iy+2w_iz+d_i=0\ (i=1,\ 2)$$

at any common point are at right angles if $2(u_1u_2+v_1v_2+w_1w_2)=d_1+d_2$. 6

(b) Find the equation of the cylinder whose generators are parallel to the x-axis and which passes through the curve of intersection of the plane $2x-3y+z=1$ and the surface $3y^2-5z^2=12x$. 6

Or

Prove that the plane $ax+by+cz=0$ cuts the cone $yz+zx+xy=0$ in perpendicular generators, if

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$$

(c) The section of the enveloping cone of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

with P as vertex by the plane $z=0$ is a circle. Find the locus of P. 6

Or

Find the equation of the polar plane of the point $(3, 7, -2)$ with respect to the conicoid $3x^2-2y^2+5z^2=4$.

GROUP—B

(Differential Equations)

(*Marks* : 36)

5. (a) Form the differential equation of the family of circles

$$x^2+y^2+2gx+2fy+c=0$$ 4

(b) The rate of change in temperature of an object varies as the difference in temperature between the object and surroundings. If an object cools from 80 °C to 60 °C in 20 minutes, find the temperature in 40 minutes if the surrounding temperature is 20 °C. 6

6. Answer any *two* parts of the following : 4×2=8

(a) Solve

$$(xy\cos xy + \sin xy)dx + x^2 \cos xy\, dy = 0$$

(b) Find the solution of the differential equation

$$(p^2 + xy) = p(x + y)$$

where $p = \frac{dy}{dx}$. Also, find the singular solution.

(c) Solve

$$x^2(xdx + ydy) + 2y(xdy - ydx) = 0$$

7. Solve any *two* parts of the following : 4×2=8

(i) $\frac{d^2y}{dx^2} + a^2y = \sec ax$

(ii) $(D^2 - 2D + 4)y = e^x \cos x$

(iii) $x\frac{dy}{dx} - y = (x-1)\left(\frac{d^2y}{dx^2} - x + 1\right)$

8. *(a)* Solve

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)}$$ 5

Or

Solve

$$\frac{d^2y}{dx^2} - 2\tan x \frac{dy}{dx} + 5y = e^x \sin x$$

(b) Solve

$$\frac{d^2x}{dt^2} + 4x + y = te^t$$

$$\frac{d^2y}{dt^2} + y - 2x = \sin^2 t$$ 5

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